EFFECTS OF RADIATION ABSORPTION AND ALIGNED MAGNETIC FIELD ON UNSTEADY CONVECTIVE FLOW ALONG A VERTICAL POROUS PLATE

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Abstract

In this article, an analysis is carried out to study the effects of aligned magnetic field, radiation absorption and viscous dissipation on the MHD unsteady convective heat and mass transfer flow of a viscous incompressible electrically conducting and heat absorbing fluid along a vertical porous plate embedded in a porous medium with variable temperature and concentration. Approximate solutions for velocity, temperature and concentration are obtained by solving the governing equations of the flow field using multi parameter perturbation technique. The expressions for skin friction, the rate of heat transfer and mass transfer from the plate to the fluid are derived in non-dimensional form. The effects of various flow parameters affecting the flow field are discussed. It is found that the velocity increases with an increasing thermal Grashof number or massGrashof number or permeability parameter or viscous dissipationwhereas it decreases for magnetic field parameter or heat sourceparameteror radiation absorption coefficient or angle ϕ. The temperature increases for increasing viscous dissipation while it decreases for increasing Prandtl number or radiation absorption coefficient in the presence of heat source parameter. A growing heat source parameter or heat sink parameter retards the temperature.

Keywords: Radiation absorption, porous medium, viscous dissipation, heat source/sink.

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1. Introduction

Flow problems through porous media over flat surfaces are of great theoretical as well as practical interest in view of their applications in various fields such as aerodynamics, extraction of plastic sheets, cooling of infinite metallic plates in a cool bath, liquid film condensation process and in major fields of glass and polymer industries. The study of heat and mass transfer with magnetic field effect is of considerable importance in chemical and hydrometallurgical industries.Soundalgekar [1] studied the viscous dissipation effects on unsteady free convective flow past an infinite vertical porous plate with constant suction. The two dimensional unsteady free convective and mass transfer flow of an incompressible viscous dissipative and electrically conducting fluid past an infinite vertical porous plate was examined by Gregantopouloset al.[2]. Kinyanjui et al. [3] solved the problem of MHD free convective heat and mass transfer of a heat generating fluid pastan impulsively started infinite vertical porous plate with Hall current and radiation absorption by using a finite difference scheme. The effect of the viscous dissipation term along with temperature dependent heat source/ sink on momentum, heat and mass transfer in a visco-elastic fluid flow over an accelerating surface was studied by Sonth et al. [4]. Cookeyet al. [5] investigated the influence of viscous dissipation and radiation on unsteady MHD free convective flow past an infinite heated vertical plate in a porous medium with time dependent suction. Aissaand Mohammadein [6] have analyzed the effects of the magnetic parameter, Joule heating, viscous dissipation and heat generation on the MHD micropolar fluids that passed through a stretching sheet.Salem [7] investigated the coupled heat and mass transfer in Darcy-Forchheimer mixed convection from a vertical flat plat embedded in a fluid-saturated porous medium under the effects of radiation and viscous dissipation. Zueco [8] used network simulation method (NSM) to study the effects of viscous dissipation and radiation on unsteady MHD free convective flow past a vertical porous plate. Prasad and Reddy [9] investigated radiation and mass transfer effects on an unsteady MHD free convective flow past a semi-infinite vertical permeable moving plate with viscous dissipation. The effects of viscous and joules dissipation on MHD flow, heat and mass transfer past a stretching porous surface embedded in porous medium were studied by Anjali Devi and Ganga [10]. HemantPoonia and Chaudary[11] haveanalyzed the heat and mass transfer flow with viscous dissipationon an unsteady mixed convective flow along a vertical plate embedded in porous mediumwith suction. The effects of thermal radiation and variable viscosity on the unsteady hydromagnetic flow of an electrically conducting fluid over a porous vertical plate in the presence of viscous dissipation and time-dependent-suction has been presented by Mahmoud[12]. Ahmed and Batin [13] presented an analytical model for MHD mixed convective radiating fluid with viscous dissipation.BalaSidduluMalga and NaikotiKishan [14] have studied the effects of viscous dissipation on unsteady free convection and mass transfer boundary layer flow past an accelerated infinite vertical porous flat plate with suction. The effect of magnetic field on unsteady free convective flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous flat plate embedded in a porous medium in the presence of constant suction and heat sink has been studied by Das et al. [15].

In this article an attempt is made to study the effects of aligned magnetic field, radiation absorption and viscous dissipation on the unsteady convective heat and mass transfer flow along a vertical porous flat surface through a porous medium with heat source/sink.

2. Mathematical Formulation

The two dimensional unsteady free convective flow of a laminar viscous incompressible electrically conducting and heat (radiation) absorbing fluid past an infinite vertical porous plate embedded in an uniform porous medium in the presence ofheat source or sink with constant suction under the action of aligned magnetic field strength B_0 has been considered. $x^{'}$-axis is taken vertically upward direction along the plate and $y^{'}$-axis normal to it as shown in figure 1.

In order to derive the fundamental equations we assume that (i) the flow variables are functions of y and t only,since the plate is infinite in extent (ii) ρ the density of the fluidto be constant (iii) the magnetic Reynolds number is small so that the induced magnetic field can be neglected (iv) the Hall effect, electrical effect and polarization effect are neglected (v) theJoule's dissipation term in the energy equation is neglected(vi) due to the application of suction at the surface, the fluid particles at the edge of the boundary layer will have a tendency to get displaced towards the plate surface, therefore $v^{'} \rightarrow -v_0^{'}$ at $y^{'} \rightarrow \infty$ and this phenomenon is clearly supported by the equation of continuity.By usingBoussinesq's approximation, the governing equations of the flow field are given by

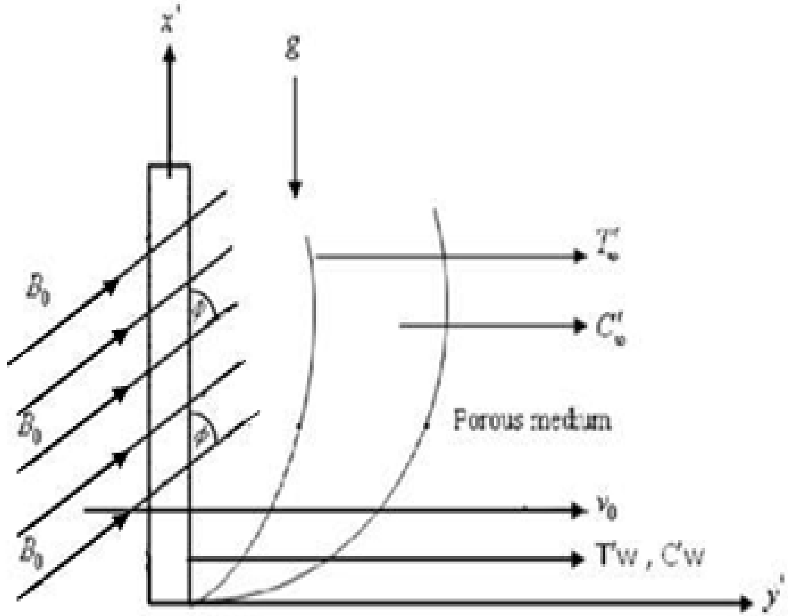

Fig 1 The flow configuration and co-ordinate system.

Equation of continuity

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v'_o \qquad (1)$$

Equation of Momentum

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \upsilon\frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) - \frac{\sigma B_0^2 Sin^2\phi}{\rho}u' - \frac{\upsilon}{k'}u' \qquad (2)$$

Equation of Energy

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{K}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{\upsilon}{\rho c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{Q_0}{\rho c_p}\left(T' - T'_\infty\right) + Q'_1\left(C' - C'_\infty\right) \qquad (3)$$

Equation of Species Diffusion

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{4}$$

The initial and boundary conditions are

$$u' = 0, v' = -v_0', T' = T_w' + \varepsilon\left(T_w' - T_\infty'\right)e^{i\omega' t'}, C' = C_w' + \varepsilon\left(C_w' - C_\infty'\right)e^{i\omega' t'} \ at \ y' = 0$$
$$u' \to 0, T' \to T_\infty', C' \to C_\infty' \ as \ y' \to \infty \tag{5}$$

Introducing the following non-dimensional quantities

$$y = \frac{y' v_0'}{\upsilon}, t = \frac{t' v_0'^2}{4\upsilon}, \omega = \frac{4\upsilon\omega'}{v_0'^2}, u = \frac{u'}{v_0'}, \upsilon = \frac{\mu}{\rho}, \theta = \frac{T' - T_\infty'}{T_w' - T_\infty'}, C = \frac{C' - C_\infty'}{C_w' - C_\infty'},$$
$$M = \left(\frac{\sigma B_0^2}{\rho}\right)\frac{\upsilon}{v_0'^2}, k = \frac{v_0^2 k'}{\upsilon^2}, Gr = \frac{\upsilon g\beta(T_w' - T_\infty')}{v_0'^3}, Gm = \frac{\upsilon g\beta^*(C_w' - C_\infty')}{v_0'^3},$$
$$\Pr = \frac{\mu c_p}{K}, Ec = \frac{v_0'^2}{c_p(T_w' - T_\infty')}, Q = \frac{4Q_0\upsilon}{\rho c_p v_0'^2}, Q_1 = \frac{\upsilon Q_1'(C_w' - C_\infty')}{(T_w' - T_\infty')v_0'^2} \tag{6}$$

in equations (2), (3) and (4) under the boundary condition (5), we get

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\,\theta + Gm\,C - \left(MSin^2\phi + \frac{1}{k}\right)u \tag{7}$$

$$\frac{1}{4}\frac{\partial\theta}{\partial t} - \frac{\partial\theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{4}Q\,\theta + \frac{1}{4}Q_1 C \tag{8}$$

$$\frac{1}{4}\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{9}$$

The corresponding boundary conditions are

$$u=0,\theta=1+\varepsilon e^{i\omega t},C=1+\varepsilon e^{i\omega t}\ at\ y=0$$
$$u\to 0,\theta\to 0,C\to 0\ as\ y\to\infty \qquad (10)$$

Here g is the acceleration due to gravity, ρ is the density, σ is the electrical conductivity, β is the coefficient of volumetric thermal expansion, β^* is the coefficient of volumetric mass expansion, $v_0^{'}$ is a constant suction velocity, ν is the coefficient of kinematic viscosity, ω is the angular frequency, μ is the coefficient of viscosity, K is the thermal diffusivity, $T^{'}$ is the temperature, $T_w^{'}$ is the temperature at the plate, $T_\infty^{'}$ is the temperature at infinity, c_p is the specific heat at constant pressure, Pr is the Prandtl number, Sc is the Schmidt number, M is the magnetic field parameter, k is the permeability parameter, Gr is the Grashof number for heat transfer, Gm is the Grashof number for mass transfer, Q is the heat source/sink parameter, Q_1 is the radiation absorption coefficient , ϕ is an align angle and Ec is the viscous dissipation or Eckert number.

3. Method of Solution

In order to solve equations (7), (8) and (9) we assume ε to be very smallandthe concentration, temperature, velocity of the flow field in the neighborhood of the plate as

$$C(y,t)=C_0(y)+\varepsilon e^{i\omega t}C_1(y) \qquad (11)$$

$$\theta(y,t)=\theta_0(y)+\varepsilon e^{i\omega t}\theta_1(y) \qquad (12)$$

$$u(y,t)=u_0(y)+\varepsilon e^{i\omega t}u_1(y) \qquad (13)$$

Substituting equations (11) to (13) in to equations (7) to (9) respectively and equating the harmonic and non-harmonic terms and neglecting the coefficients of ε^2, we get

Zeroth order equations

$$C_0^{''} = 0 \tag{14}$$

$$\theta_0^{''} + \Pr \theta_0^{'} + \frac{\Pr Q \theta_0}{4} = \frac{-\Pr Q_1 C_0}{4} - \Pr\ Ec\ u_0^{'2} \tag{15}$$

$$u_0^{''} + u_0^{'} - (M\ Sin^2\phi + \frac{1}{k}) u_0 = -Gr\,\theta_0 - Gm\ C_0 \tag{16}$$

First order equations

$$C_1^{''} - \frac{Sc\,i\omega}{4} C_1 = 0 \tag{17}$$

$$\theta_1^{''} + \Pr \theta_1^{'} + \left(\frac{Q - i\omega}{4}\right) \Pr \theta_1 = \frac{-\Pr Q_1 C_1}{4} - 2\Pr Ec\ u_0^{'}\, u_1^{'} \tag{18}$$

$$u_1^{''} + u_1^{'} - (M\ Sin^2\phi + \frac{1}{k} + \frac{i\omega}{4}) u_1 = -Gr\,\theta_1 - Gm C_1 \tag{19}$$

The corresponding boundary conditions are

$$y = 0\ :\ u_0 = 0,\ \theta_0 = 1,\ C_0 = 1,\ u_1 = 0,\ \theta_1 = 1,\ C_1 = 1$$

$$y \to \infty :\ u_0 = 0,\ \theta_0 = 0, C_0 = 0,\ u_1 = 0,\ \theta_1 = 0,\ C_1 = 0 \tag{20}$$

Equations (14)-(19) are non-linear differential equations.Using multi parameter perturbation technique and choosing Ec<<1, we assume

$$C_0 = C_{00} + Ec\ C_{01} \tag{21}$$

$$\theta_0 = \theta_{00} + Ec\ \theta_{01} \tag{22}$$

$$u_0 = u_{00} + Ec\ u_{01} \tag{23}$$

$$C_1 = C_{10} + Ec\ C_{11} \tag{24}$$

$$\theta_1 = \theta_{10} + Ec\ \theta_{11} \tag{25}$$

$$u_1 = u_{10} + Ec\ u_{11} \tag{26}$$

Now using the equations (21) to (26) in to equations (14) to (19) and equating the coefficients of like powers of Ec, neglecting those of Ec^2 because Eckert number Ec is very small for incompressible fluid flows, we get the following set of differential equations.

Zeroth order equations

$$C_{00}'' = 0 \tag{27}$$

$$C_{10}'' - \frac{Sc\, i\omega}{4} C_{10} = 0 \tag{28}$$

$$\theta_{00}'' + \Pr \theta_{00}' + \frac{\Pr Q}{4}\theta_{00} = \frac{-\Pr Q_1}{4} C_{00} \tag{29}$$

$$\theta_{10}'' + \Pr \theta_{10}' + (Q - i\omega)\frac{\Pr}{4}\theta_{10} = \frac{-\Pr Q_1}{4} C_{10} \tag{30}$$

$$u_{00}'' + u_{00}' - \left(M\, Sin^2\phi + \frac{1}{k}\right)u_{00} = -Gr\ \theta_{00} - Gm\ C_{00} \tag{31}$$

$$u_{10}'' + u_{10}' - \left(M\, Sin^2\phi + \frac{1}{k} + \frac{i\omega}{4}\right)u_{10} = -Gr\ \theta_{10} - Gm\ C_{10} \tag{32}$$

The corresponding boundary conditions are

$$\begin{aligned} &y = 0 \quad : u_{00} = 0, \theta_{00} = 1, C_{00} = 1, u_{10} = 0, \theta_{10} = 1, C_{10} = 1 \\ &y \to \infty : \ u_{00} = 0, \theta_{00} = 0, C_{00} = 0, u_{10} = 0, \theta_{10} = 0, C_{10} = 0 \end{aligned} \tag{33}$$

First order equations

$$C_{01}'' = 0 \tag{34}$$

$$C_{11}'' - \frac{Sc\, i\omega}{4} C_{11} = 0 \tag{35}$$

$$\theta_{01}'' + \Pr \theta_{01}' + \frac{\Pr Q}{4}\theta_{01} = \frac{-\Pr Q_1}{4} C_{01} - \Pr u_{00}'^2 \tag{36}$$

$$\theta_{11}^{''} + \Pr\theta_{11}^{'} + (Q - i\omega)\frac{\Pr}{4}\theta_{11} = \frac{-\Pr Q_1}{4}C_{11} - 2\Pr u_{00}^{'}u_{10}^{'} \tag{37}$$

$$u_{01}^{''} + u_{01}^{'} - \left(M\ Sin^2\phi + \frac{1}{k}\right)u_{01} = -Gr\ \theta_{01} - Gm\ C_{01} \tag{38}$$

$$u_{11}^{''} + u_{11}^{'} - \left(M\ Sin^2\phi + \frac{1}{k} + \frac{i\omega}{4}\right)u_{11} = -Gr\,\theta_{11} - Gm\,C_{11} \tag{39}$$

The corresponding boundary conditions are

$$\begin{aligned} y = 0 \quad &: u_{01} = 0, \theta_{01} = 0, C_{01} = 0, u_{11} = 0, \theta_{11} = 0, C_{11} = 0 \\ y \to \infty &: u_{01} = 0, \theta_{01} = 0, C_{01} = 0, u_{11} = 0, \theta_{11} = 0, C_{11} = 0 \end{aligned} \tag{40}$$

The ordinary differential equations (27) to (32) and (34) to (39) aresolved subject to the boundary conditions (33) and (40) respectively. Then substituting the solutions in to equations (21) to (26), we obtained the exact solutions for concentration, temperature and velocity as follows:

$$C(y,t) = 1 + \varepsilon\, e^{i\omega t}\, e^{-m_1 y} \tag{41}$$

$$\begin{aligned} \theta(y,t) = {} & [(A_4 e^{-m_3 y} - A_3) + Ec\,(A_{28}\, e^{-2m_5 y} + A_{29}\, e^{-2m_3 y} + A_{30}\, e^{-(m_3+m_5)y} + A_{31}\, e^{-m_3 y})] + \\ & \varepsilon\, e^{i\omega t}[(A_2\, e^{-m_2 y} - A_1 e^{-m_1 y}) + Ec(A_{21}\, e^{-(m_4+m_5)y} + A_{22}\, e^{-(m_3+m_4)y} + \\ & A_{23}\, e^{-(m_2+m_5)y} + A_{24}\, e^{-(m_2+m_3)y} + A_{25}\, e^{-(m_1+m_5)y} + A_{26}\, e^{-(m_1+m_3)y} + A_{27}\, e^{-m_2 y}] \end{aligned} \tag{42}$$

$$\begin{aligned} u(y,t) = {} & [(A_{10}\, e^{-m_3 y} + A_{13} + A_{14}\, e^{-m_5 y}) + Ec\,(A_{40}\, e^{-2m_5 y} + A_{41}\, e^{-2m_3 y} + A_{42}\, e^{-(m_3+m_5)y} + A_{43}\, e^{-m_3 y} + A_{44}\, e^{-m_5 y})] \\ & + \varepsilon\, e^{i\omega t}[(A_5\, e^{-m_2 y} + A_8 e^{-m_1 y} + A_9\, e^{-m_4 y}) + Ec(A_{32}\, e^{-(m_4+m_5)y} + A_{33}\, e^{-(m_3+m_4)y} + A_{34}\, e^{-(m_2+m_5)y} + \\ & A_{35}\, e^{-(m_2+m_3)y} + A_{36}\, e^{-(m_1+m_5)y} + A_{37}\, e^{-(m_1+m_3)y} + A_{38}\, e^{-m_2 y} + A_{39}\, e^{-m_4 y})] \end{aligned}$$

(43)

4. Skin Friction

Skin-friction coefficient τ at the plate is given by

$$\tau = \left[\frac{\partial u}{\partial y}\right]_{y=0} \tag{44}$$

Using equations (43) and (44), we obtained the skin – friction at the plate in non –dimensional form as follows

$$\begin{aligned}\tau = {} & [(-m_3 A_{10} - m_5 A_{14}) + Ec(-2m_5 A_{40} - 2m_3 A_{41} - (m_3 + m_5)A_{42} - m_3 A_{43} - m_5 A_{44})] + \\ & \varepsilon\, e^{i\omega t}[(-m_1 A_8 - m_2 A_5 - m_4 A_9) + Ec(-(m_4 + m_5)A_{32} - (m_3 + m_4)A_{33} - (m_2 + m_5)A_{34} - \\ & (m_2 + m_3)A_{35} - (m_1 + m_5)A_{36} - (m_1 + m_3)A_{37} - m_2 A_{38} - m_4 A_{39})]\end{aligned} \tag{45}$$

5. Nusselt Number

The rate of heat transfer coefficient Nu at the plate is given by

$$Nu = \left[\frac{\partial \theta}{\partial y}\right]_{y=0} \tag{46}$$

Using equations (42) and (46), we obtained Nusselt number in non –dimensional form as follows

$$\begin{aligned}Nu = {} & [-m_3 A_4 + Ec(-2m_5 A_{28} - 2m_3 A_{29} - (m_3 + m_5)A_{30} - m_3 A_{31})] + \\ & \varepsilon\, e^{i\omega t}[(m_1 A_1 - m_2 A_2) + Ec(-(m_4 + m_5)A_{21} - (m_3 + m_4)A_{22} - (m_2 + m_5)A_{23} - \\ & (m_2 + m_3)A_{24} - (m_1 + m_5)A_{25} - (m_1 + m_3)A_{26} - m_2 A_{27})]\end{aligned} \tag{47}$$

6. Sherwood Number

The rate of mass transfer coefficient Sh at the plate is given by

$$Sh = \left[\frac{\partial c}{\partial y}\right]_{y=0} \tag{48}$$

Using equations (41) and (48), we obtained Sherwood number in non –dimensional form as follows

$$Sh = -m_1 \varepsilon\, e^{i\omega t} \tag{49}$$

Where $M_1 = MSin^2\phi + \frac{1}{k}, M_2 = MSin^2\phi + \frac{1}{k} + \frac{i\omega}{4}, M_3 = (Q - i\omega)\frac{\Pr}{4}, M_4 = \frac{\Pr Q}{4},$

$$A_1 = \frac{\Pr Q_1}{4(m_1^2 - \Pr m_1 + M_3)}, A_2 = 1 + A_1, A_3 = \frac{Q_1}{Q}, A_4 = 1 + A_3, A_5 = \frac{-Gr\, A_2}{m_2^2 - m_2 - M_2}, A_6 = \frac{Gr\, A_1}{m_1^2 - m_1 - M_2},$$

$$A_7 = \frac{Gm}{m_1^2 - m_1 - M_2}, A_8 = A_6 - A_7, A_9 = -(A_5 + A_8), A_{10} = \frac{-Gr\, A_4}{m_3^2 - m_3 - M_1}, A_{11} = \frac{Gr\, A}{M_1}, A_{12} = \frac{Gm}{M_1},$$

$$A_{13} = A_{12} - A_{11},\ A_{14} = -(A_9 + A_{10}),\ A_{15} = A_9 A_{14} m_4 m_5, A_{16} = A_9 A_{10} m_3 m_4, A_{17} = A_5 A_{14} m_2 m_5, A_{18} = A_5 A_{10} m_2 m_3,$$

$$A_{19} = A_8 A_{14} m_1 m_5,\ A_{20} = A_8 A_{10} m_1 m_3, A_{21} = \frac{-2\Pr A_{15}}{(m_4 + m_5)^2 - \Pr(m_4 + m_5) + M_3},$$

$$A_{22} = \frac{-2\Pr A_{16}}{(m_3 + m_4)^2 - \Pr(m_3 + m_4) + M_3}, A_{23} = \frac{-2\Pr A_{17}}{(m_2 + m_5)^2 - \Pr(m_2 + m_5) + M_3},$$

$$A_{24} = \frac{-2\Pr A_{18}}{(m_2 + m_3)^2 - \Pr(m_2 + m_3) + M_3},\ A_{25} = \frac{-2\Pr A_{19}}{(m_1 + m_4)^2 - \Pr(m_1 + m_5) + M_3},$$

$$A_{26} = \frac{-2\Pr A_{20}}{(m_1 + m_3)^2 - \Pr(m_1 + m_3) + M_3}, A_{27} = -(A_{21} + A_{22} + A_{23} + A_{24} + A_{25} + A_{26}),$$

$$A_{28} = \frac{-\Pr m_5^2 A_{14}^2}{4m_5^2 - 2m_5 \Pr + M_4},\ A_{29} = \frac{-\Pr m_3^2 A_{10}^2}{4m_3^2 - 2m_3 \Pr + M_4}, A_{30} = \frac{-2\Pr m_3 m_5 A_{10} A_{14}}{(m_3 + m_5)^2 - \Pr(m_3 + m_5) + M_4},$$

$$A_{31} = -(A_{28} + A_{29} + A_{30}),\ A_{32} = \frac{-Gr\, A_{24}}{(m_2 + m_3)^2 - (m_2 + m_3) - M_2},\ A_{33} = \frac{-Gr\, A_{25}}{(m_1 + m_5)^2 - (m_1 + m_5) - M_2},$$

$$A_{34} = \frac{-Gr\, A_{26}}{(m_1 + m_3)^2 - (m_1 + m_3) - M_2}, A_{35} = \frac{A_{27}}{m_2^2 - m_2 - M_2}, A_{36} = \frac{-Gr\, A_{21}}{(m_4 + m_5)^2 - (m_4 + m_5) - M_2},$$

$$A_{37} = \frac{-Gr\, A_{22}}{(m_3 + m_4)^2 - (m_3 + m_4) - M_2},\ A_{38} = \frac{-Gr\, A_{23}}{(m_2 + m_5)^2 - (m_2 + m_5) - M_2},$$

$$A_{39} = -(A_{32} + A_{33} + A_{34} + A_{35} + A_{36} + A_{37} + A_{38}),\ A_{40} = \frac{-Gr\, A_{28}}{4m_5^2 - 2m_5 - M_1},\ A_{41} = \frac{-Gr\, A_{29}}{4m_3^2 - 2m_3 - M_1},$$

$$A_{42} = \frac{-Gr\, A_{30}}{(m_3 + m_5)^2 - (m_3 + m_5) - M_1},\ A_{43} = \frac{-Gr\, A_{31}}{m_3^2 - m_3 - M_1},\ A_{44} = -(A_{40} + A_{41} + A_{42} + A_{43})$$

7. Results and Discussion

In order to get an insight in the physical situation of the problem, the numerical values of the velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number at the plate are obtained for different values of the physical parameters involved in the flow field and are analyzed with the help of figures from 2 to 13. The value of *Sc* is taken to be 0.66 which corresponds to water-vapor and *Pr* is taken to be 0.71 which corresponds to air at $25^0 C$ temperature with one atmospheric pressure. ωt is taken to be $\pi/2$ and $\phi$is taken to be $\pi/6$.The values of the other physical parameters are chosen arbitrarily.

The concentration profile is plotted for different values of Schmidt number *Sc* in figure 2. It is observed that the effect of increasing values of *Sc* is to decrease the concentration.The variation of the temperature distribution for various values of *Pr*, *Ec*, *Q* and Q_1 are represented in figures 3 to 6. Figure 3 shows the effect of Prandtl number *Pr* on the temperature distribution. It is obvious that with the increase in the values of *Pr*, the temperature across the boundary layer decreases. Figure 4 illustrates the influence of the viscous dissipation *Ec* on the temperature profile in the boundary layer with respect to heat source parameter. It has been observed that as *Ec* increases, the temperature increases. From numerical calculations the same trend is noticed in the case of heat sink parameter (Q<0).

Figure 5depicts the dimensionless temperature profiles for different values of heat source parameter (Q>0).It is noted that increasing the values of heat source parameter causes a reduction in the fluid temperature.From numerical calculations the same phenomenon is observed in the case of heat sink parameter (Q<0). Figure 6shows the effect of radiation absorption coefficient Q_1 on the temperature field due toheat source parameter. It is observed that the temperature decreases as Q_1 increases. From numerical calculations a reverse trend is noticed in the case of heat sink parameter.

Figures 7 to 13 indicate the variations of the non-dimensional velocity for various flow parameters respectively. In figure 7, the velocity profile is plotted for various values of thermal Grashof number *Gr*. It is observed that the main stream velocity increases with an increase in the thermal Grashof number *Gr*.From numerical calculations, the same trend is noticed with the effect of mass Grashof number *Gm*.The influence of the magnetic field parameter M on velocity profile is predicted in figure 8. It shows that a growing magnetic field parameter retards the velocity of the flow field at all points. The effect of permeability parameter *k* is studied and the results are exhibited in figure 9. It is observed that the velocity increases with increasingpermeability parameter *k*. Figure 10 depicts the effect of an angle ϕ on the velocity field. The magnitude of the velocity decreases with increase of angle ϕ.

The variation of the velocity profile is shown in the figure 11 for varying values of viscous dissipation *Ec*with respect to heat source parameter. The velocity increases with an increase of *Ec*.Figure 12 is a graphical representation which depicts the velocity distribution for different values of heat source parameter. It is clear that, increasing heat source parameter retards the velocity of the flow field at all points.The influence of radiation absorption coefficient Q_1 on the velocity profile is illustrated in figure 13 in the presence of heat source parameter. It is observed that the velocity decreases with an increase of Q_1.

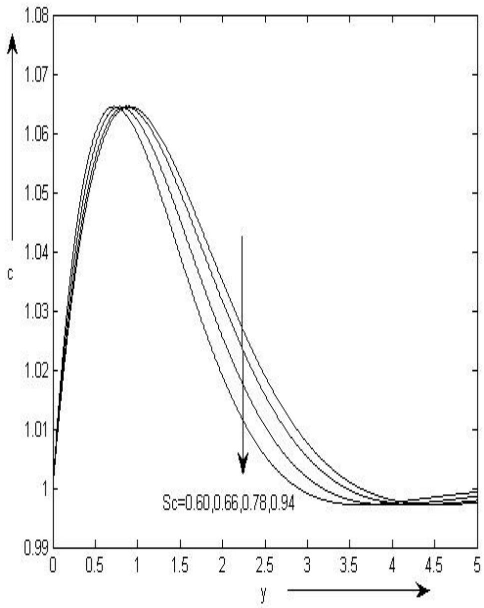

Fig.2 Effect of *Sc* on concentration field when ω=5.0, ε=0.2

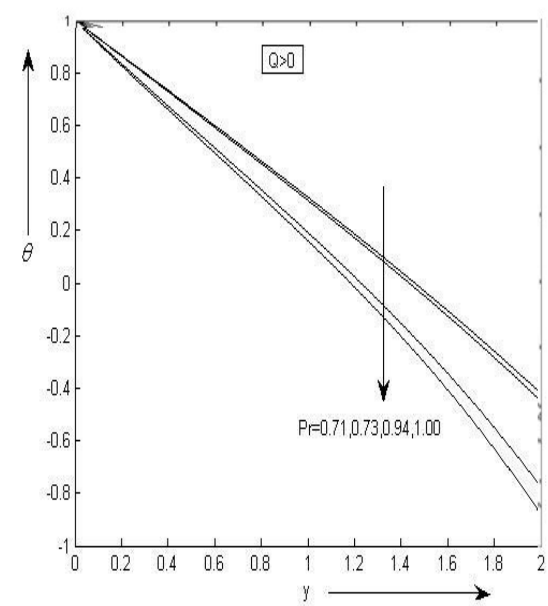

Fig. 3 Effect of *Pr* on temperature field when *Gr*=5, *Gm*=1, *M*=1, *Ec*=0.002, *K*=1, *Q*=1, Q_1=0.5, ε=0.2, ω=1.0

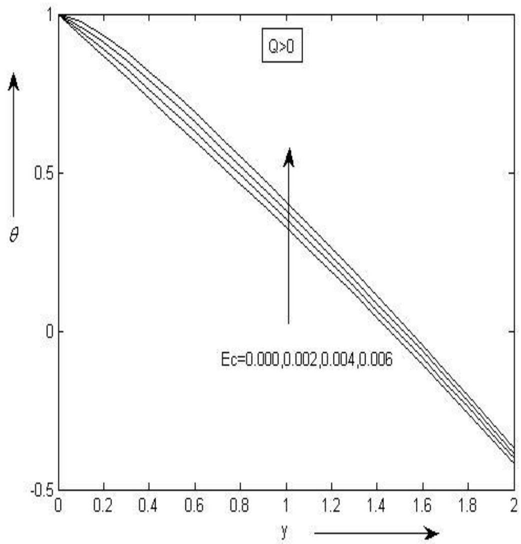

Fig. 4 Effect of *Ec* on temperature field when *Gr*=10, *Gm*=10, *M*=1, *k*=1, *Q*=1, Q_1=0.5, ε=0.2, ω=1.0

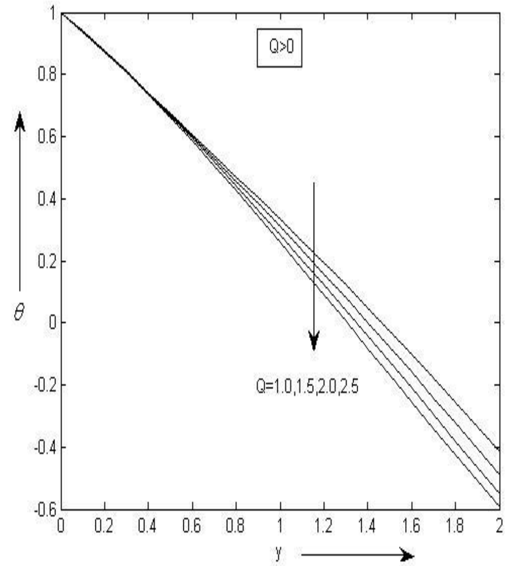

Fig. 5 Effect of *Q* (for heat source parameter) on Temperature field when *Gr*=5, *Gm*=5, *Ec*=0.002, *M*=1, *k*=1, Q_1=0.5, ε=0.2, ω=1.0

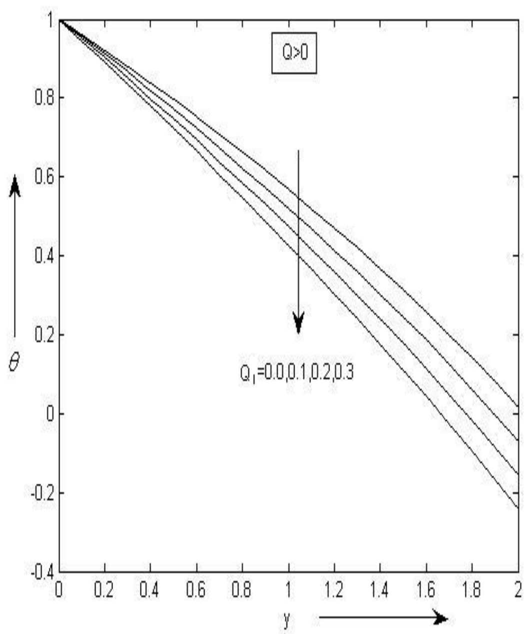

Fig. 6 Effect of Q_1 on temperature field when *Gr*=5, *Gm*=5,*Ec*=0.002 *M*=1, *k*=5, *Q*=1, ε=0.2, ω=1.0

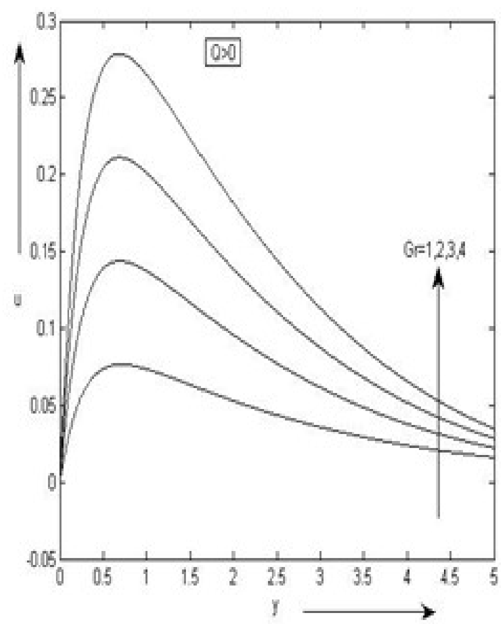

Fig. 7 Effect of *Gr* on velocity field when *Gm*=0.1, *Ec*=0.001, *M*=1, *k*=0.1,*Q*=1,Q_1=0.01, ε=0.1, ω=1.0

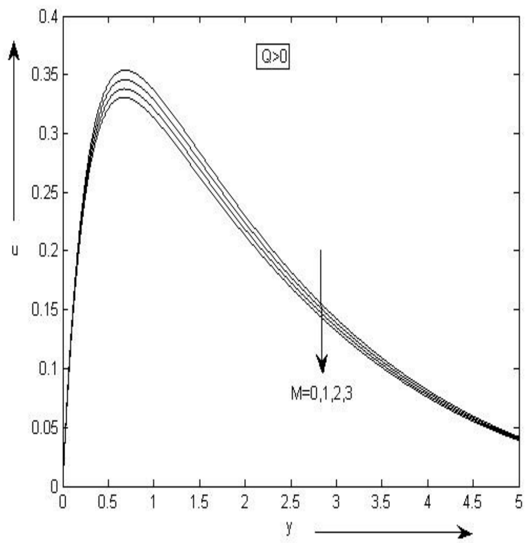

Fig. 8 Effect of M on velocity field when Gr=5, Gm=0.1, Ec=0.001,k=0.1,Q=1, Q_1=0.01,ε=0.1, ω=1.0

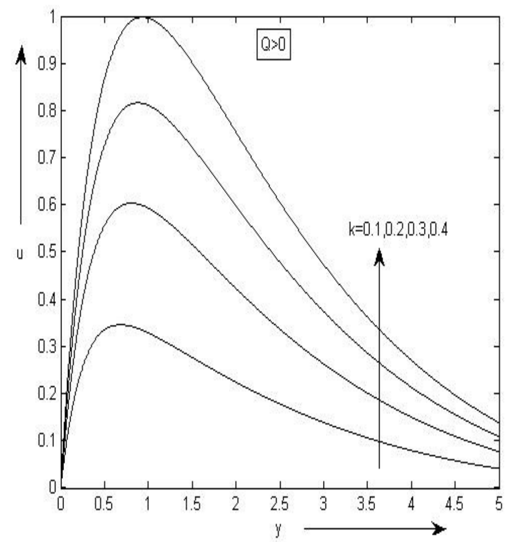

Fig. 9 Effect of k on velocity field when Gr=5, Gm=0.1, Ec=0.001,M=1,Q=1, Q_1=0.01,ε=0.1, ω=1.0

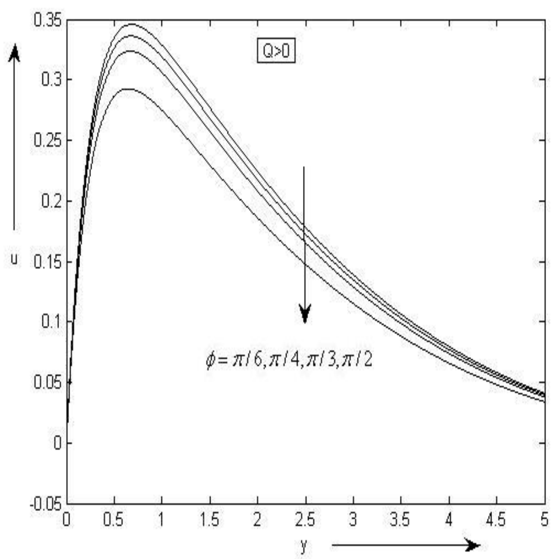

Fig. 10 Effect of ϕ on velocity field when Gr=5, Gm=0.1, Ec=0.001,M=1, k=0.1,Q=1, Q_1=0.01,ε=0.1, ω=1.0

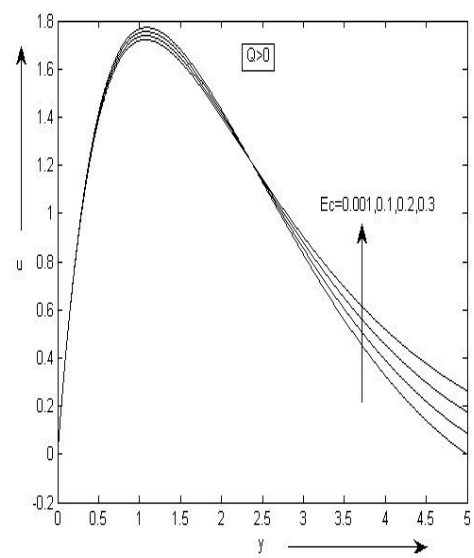

Fig. 11 Effect of Ec on velocity field when Gr=5, Gm=0.1, M=1, k=1,Q=1, Q_1=0.01,ε=0.1, ω=1.0

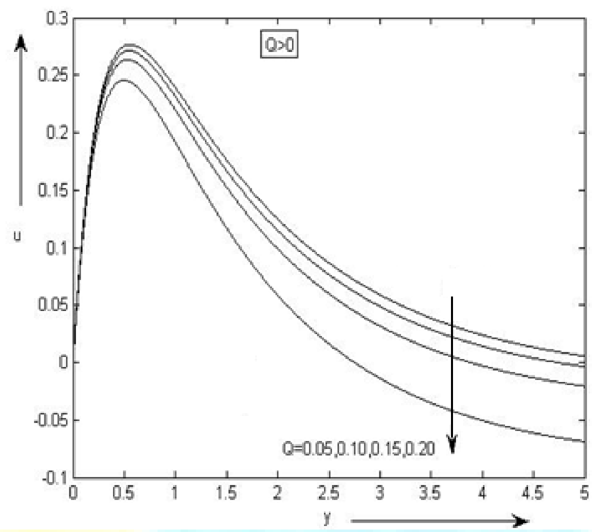

Fig. 12 Effect of Q (for heat source parameter) on velocity field when Gr=5, Gm=0.1, M=1, k=0.1,Q_1=0.01,ε=0.1, ω=1, Ec=0.001

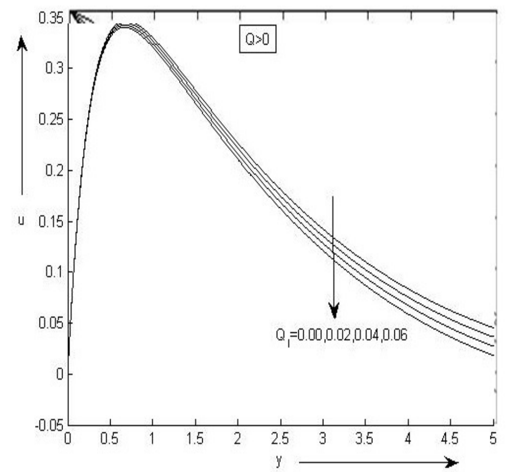

Fig. 13 Effect of Q_1on velocity field when Gr=5, Gm=0.1, M=1, k=0.1,Q=1,ε=0.1, ω=1, Ec=0.001

8. Conclusions

Based on the results and discussions, the following conclusions have been arrived at. Increasing the Schmidt number induces reduction in the concentration andrises the rate of mass transfer and skin friction coefficient. The velocity and skin friction increase for the increase of Grashof number for heat and mass transfer or permeability parameter and decreases for the increase of magnetic field parameter or Prandtl number or angle ϕ . With the increase of heat source parameter; the velocity, temperature and skin friction decrease while the rate of heat transfer increases.Also for the increase of heat sink parameter the velocity and temperature increasewhereas the skin friction and the rate of heat transfer decrease. The velocity, temperature, skin friction and the rate of heat transfer decrease with increase in radiation absorption coefficient in the case when Q>0. But the trend is just reversed in the case when Q<0.The temperature, velocity, skin friction and the rate of heat transferincrease with the increase in viscous dissipation Ec in bothheat source and heat sink parameters.

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